Reconstructing the Magnetic field of the Milky way via Astrophysical Techniques and Numerical Simulations (MAGMASIM)

Vagelis Harmandaris[1], Pavlidou Vasiliki[2], Georgios E. Pavlou[1,2,#]
Alexandros Tsouros[2]
1. Institute of Applied and Computational Mathematics, FORTH
2. Institute of Astrophysics, FORTH
Presenting author

16 July 2022

Outline of the talk

- Introduction
- Reconstruction / Calculation strategy
- Initial Results
- Conclusions and Perspectives

Introduction: Scope of the project

- ***Aim of the Project***

↓

Reconstructing the magnetic field of our Galaxy (GMF)

Synergy between the *Institute of Applied and Computational Mathematics (IACM)* and

the *Institute of Astrophysics* (IA) of FORTH

What is the "reconstruction"?

Using

- ➢ constraints from experimental / observational measurements of the Galactic Magnetic Field and
- ➢ modern numerical tools

calculate the GMF in the all points in a geometry of interest (cone)

Introduction: Galactic Magnetic Field

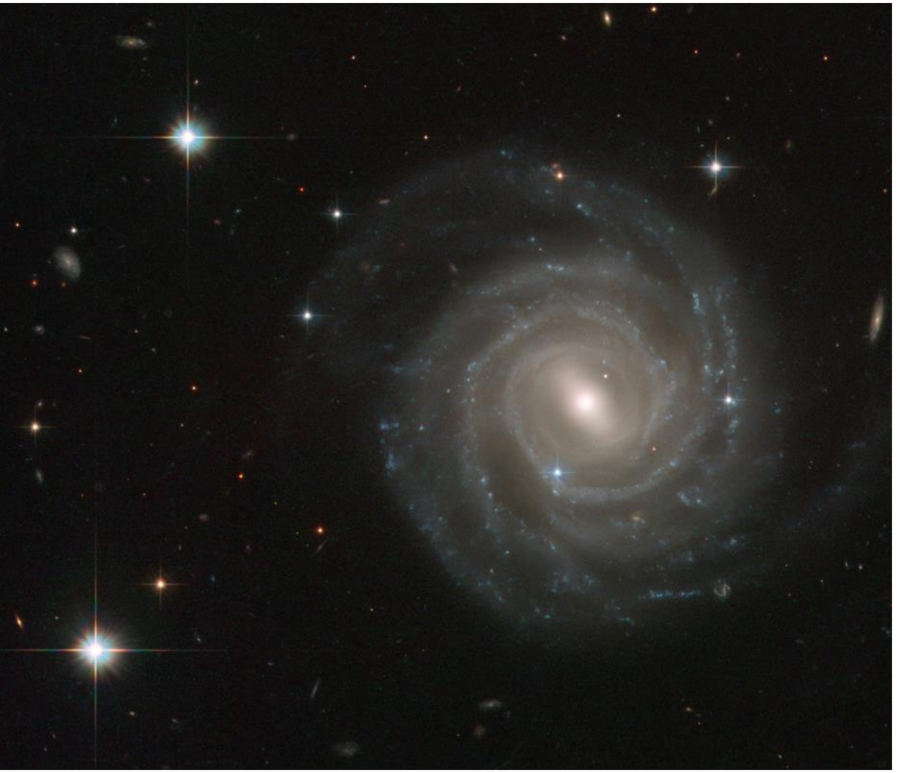
Why the study of GMF is important?

- Affects *Star Formation*
- Affects the *trajectory of high-energy cosmic rays* $\sim 10^{20} eV$ (charged high-energy particles) that give us information about high-energy astrophysical phenomena
- May give us information for the *first moments of the universe*

Basic Theoretical Tool for Magnetic Field Calculations

- *Maxwell Differential Equations* with *Boundary Conditions* (Values of the MF in the boundaries of the geometry of the calculation)
- They reduce to the Poisson equation (MF is time independent – no sources)

Introduction: Our Galaxy (Milky Way)



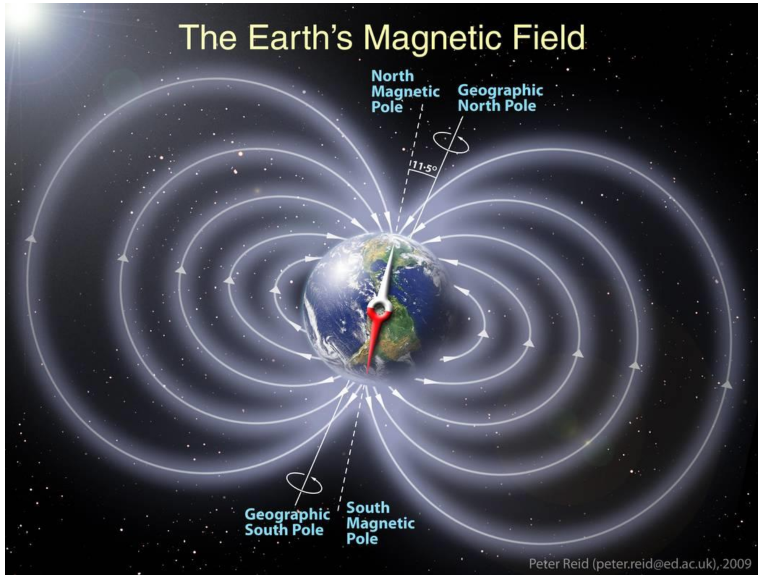
The structure of the Milky Way is thought to be similar to this galaxy (image by Hubble)

1 Light Year = $9.46 \times 10^{15} m$
1 parsec= $3.086 \times 10^{16} m$

- *Spiral Galaxy*
- *Radius*: 130000 light years
- *Total Mass*: 1.5 trillion Solar Masses
- *Contains* between 100 and 400 billion stars and at least that many planets
- The *Galactic disc* consists of a stellar component (composed of most of the galaxy's stars) and a gaseous component (mostly composed of cool gas and dust)
- The Galactic disk is surrounded by a *spheroidal halo* of old stars
- In the *Galactic Center* there is central object thought to be a supermassive Black Hole
- The *Sun* is ~8.5kpc away from the Galactic Center
- Permeated by Magnetic Field

Introduction: Galactic Magnetic Field

Magnetic fields are prevalent in the Milky Way



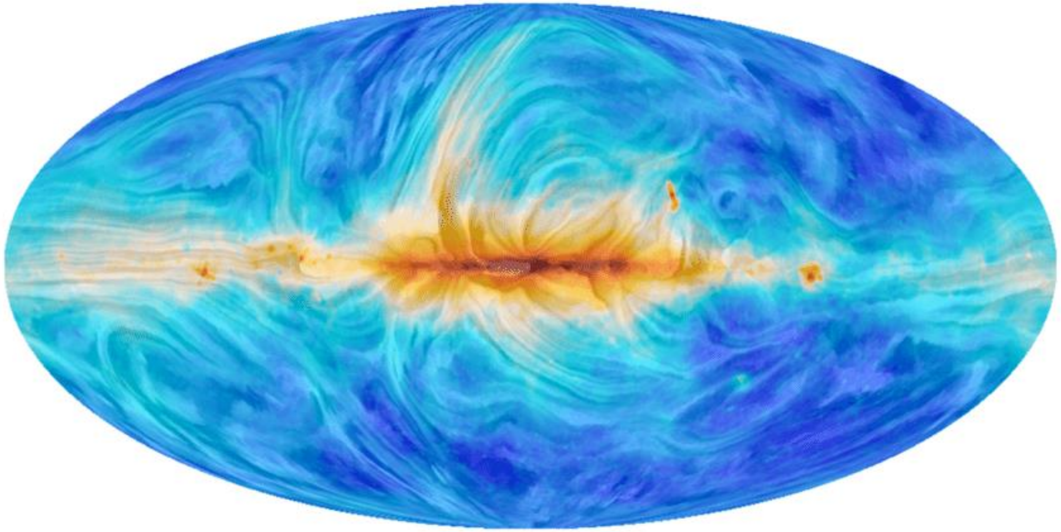


Image by NASA
MFs are mostly Galactic (stars, planets relatively small scales) (i.e. Earth's $\sim 0.5G$)

Image by Planck Telescope
The GMF is of larger scale
Observations: $\sim 6\mu G$ near the Sun, $\sim 30\mu G$ as we approach the Galactic center

Calculation Strategy: GMF Reconstruction

Numerical Simulation (solution of the Maxwell Equations) subject to constrains from experimental measurements

- Till now: Integral measurements along the line of sight (not trivial to include in calculations)
- Direct measurements of the GMF are only now starting to become available and an influx of them is expected in 5 to 10 years

State of the art: Assume a geometry for the GMF and perform a best-fit that reproduces the integral measurements

Such models are too simple to capture the full complexity of the GMF

GMF Reconstruction - Challenges

Numerical challenges:

- *Observational measurements* will be used as ***boundary conditions*** (BC) or ***sparse known data points*** in the computational grid
- However, direct measurements of the GMF are only possible at the locations of interstellar clouds → no experimental control over them. ***The GMF will be known on an irregular and very sparse grid***
- Integral constrains from the *line-of-sight* measurements must be included in the model

GMF Reconstruction – Tackling the problem

To overcome the previous challenges, our approach involves various stages

- ***The direct (forward) problem***: We solve numerically the differential equations, describing the problem, in a 3D geometry given specific boundary conditions (BCs)
- ***The Sparse Data case – Inverse problem:*** From the data created in the previous step we generate simulated direct observational data, and we try to infer the BCs by solving the inverse problem
- Inclusion of ***Integral Constrains*** that simulate the along the line-of-sight measurements
- Application to real ***direct observational data***

GMF Reconstruction – Methodology – Results

First Stage: **“The direct – forward problem”**

- A ***cone geometry*** is used to model the realistic geometry of the experimental observations
- Solve the differential equations with **modern tools** (finite elements)

↓

We use *FEniCS*

- A popular open-source computing platform for solving partial differential equations
- Enables users to quickly translate scientific models into efficient finite element code in Python

GMF Reconstruction – Results

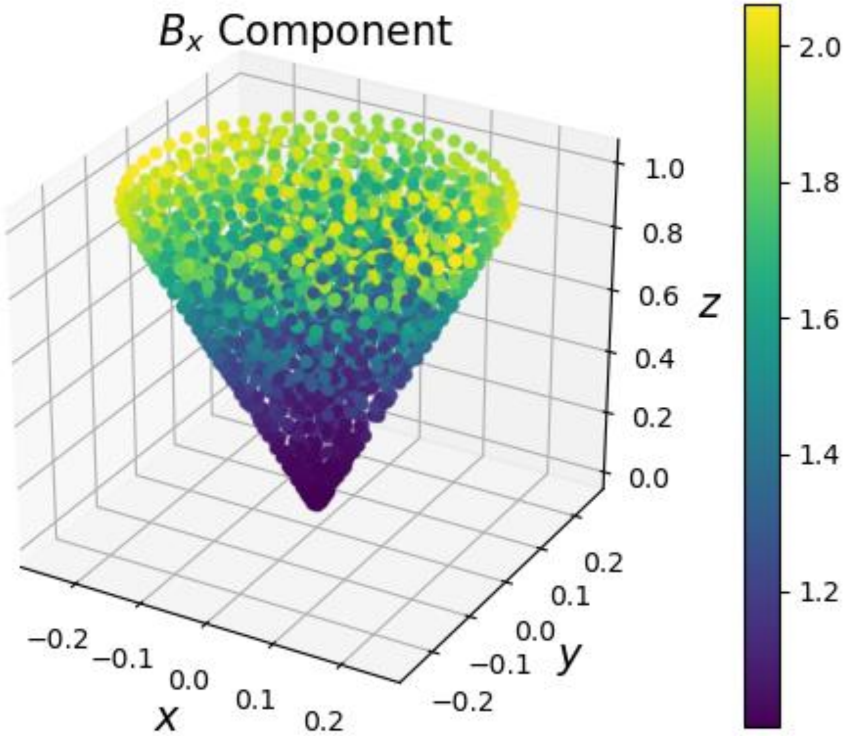
First Stage: **“The direct – forward problem”**

An example calculation

- *Poisson equation*:
  $\nabla^2 \mathbf{B} = 0,\ \mathbf{x} \in$ in cone
- *Analytical boundary conditions*
  $B_x = 1 + x^2 - 2y^2 + z^2$

(similar functions for the other components)

- *Reference calculation*: we will use this calculation it to **simulate artificial experimental data**



An example plot of the x-component of the Magnetic Field. Other components produce similar results

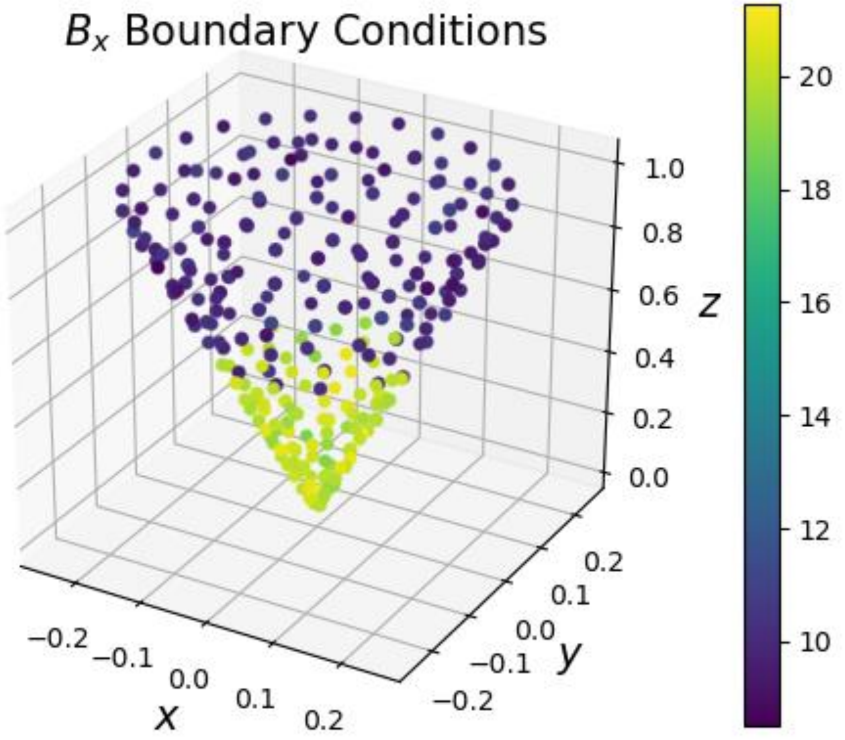
GMF Reconstruction – Results

Second Step: **“The inverse problem”**

- Consider a set **sparse direct measurements** of the MF (created from the solution of the direct problem or, in the future, from actual observational data)
- From them we want to **find information for the statistical properties of the boundary conditions**

This defines the inverse problem

- As an example, consider the *simulated* data calculated by solving the direct problem using the shown boundary conditions
- The BC are created by two normal distributions one for the upper and one for the lower half of the cone with different mean values



GMF Reconstruction – Results – A two prior example

Second Step: **“The inverse problem”**

To infer information for the boundary conditions we use **Bayesian Analysis**

$$\boldsymbol{p(\theta|y) \propto p(y|\theta)p(\theta)}$$

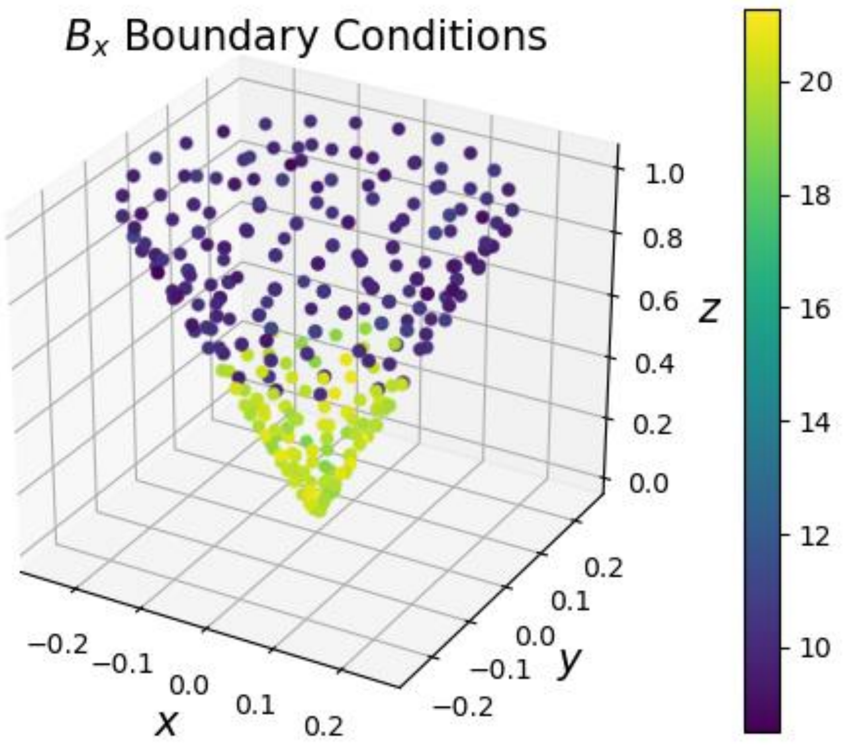
$\boldsymbol{p(\theta)}$: Prior(s), 2 in this example, distributions that describe the statistical properties of the BC (here we consider normal distributions).

The **unknown** parameters $\boldsymbol{\theta}$ can be mean values of the distributions etc.

Using values generated from the priors as BC we solve the direct problem with *FEniCS*

$\boldsymbol{p(y|\theta)}$: Likelihood, usually a normal distribution (y is the difference between this and the simulated data set)

$\boldsymbol{p(\theta|y)}$: Posterior

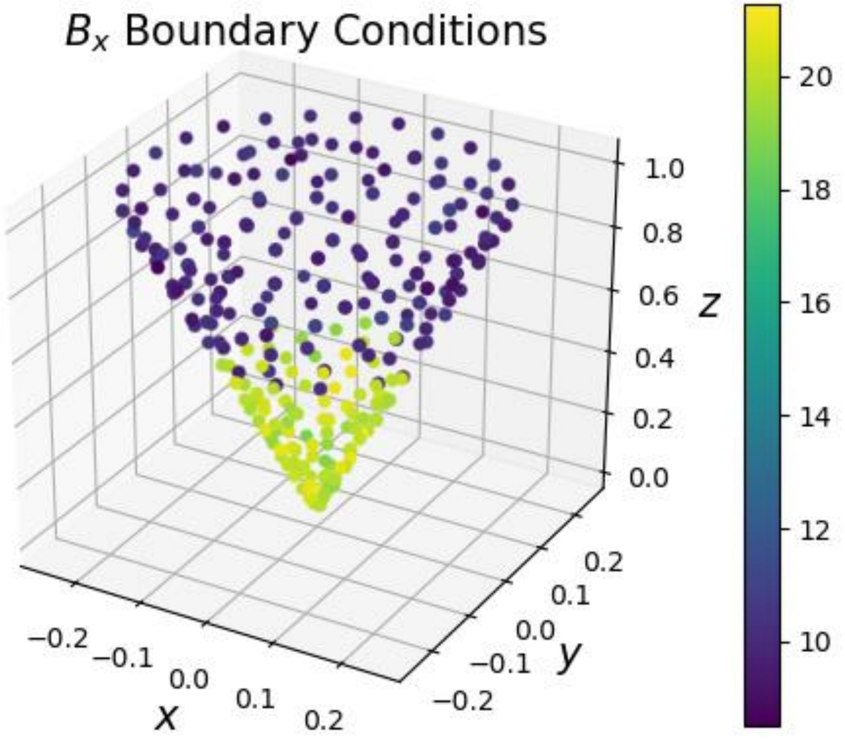


GMF Reconstruction – Results – A two prior example

Second Step: **“The inverse problem”**

- To find $\boldsymbol{\theta}$ we define and solve (with proper optimization numerical methods like simulated annealing) a *Maximum Likelihood Estimation* problem:

$$\theta_{\mathrm{MLE}} = \arg\max_{\theta}(p(\theta \mid y))$$



In this example, with the help of 2 prior distributions, the statistical properties of the initial distributions are successfully recovered

GMF Reconstruction – Current Work

- Complete the study of the **inverse problem**
  - Study the case of more priors
  - See how the sparsity of the data affects the results

- Incorporate the information from the line-of-sight measurements. This is done with the introduction of **integral constrains** in the calculation

- *Future*: Apply the inverse problem algorithm with actual **observational data** once available

Conclusions and Perspectives

- ✓The problem of the numerical reconstruction of the GMF is very interesting from both the Astrophysical and the Numerical point of view

- ✓The reconstruction will be very useful for the study of all astrophysical phenomena affected by Galactic magnetism

- ✓The problem, from the numerical point of view, is very challenging but also very interesting mainly due to sparse data and integral constrains

Thank you for you attention